Math 3210

Tutorial 10

Example 1: Some clarification from last tutorial

Solve the following LPPs by dual simplex method and find out the optimal values of all the primal and dual variables

$$\min \quad 2x_1 + x_2 + x_3$$

$$x_1 + x_2 \leq 3$$
$$x_1 - 2x_2 \geq 1$$
$$x_2 + x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0$$

the safest way is to switch it to Canonical from

$\max -2x_1 - x_2 - x_3$
$x_1 + x_2 \leq 3$
$-x_1 + x_2 \leq -1$
$-x_2 - x_3 \leq -4.$

standard from.

(1)	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	1	1	0	1	0	0	3
x_5	-1	2	0	0	1	0	-1
x_6	0	-1	−1	0	0	1	-4
x_0	2	1	1	0	0	0	0

← most negative. x_6 leaving.

ratio $-\frac{x_0}{x_6}$: $-\infty$, -1, $\boxed{-1}$ → x_3 entering.

(2)	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	1	1	0	1	0	0	3
x_5	−1	2	0	0	1	0	-1
x_3	0	1	1	0	0	-1	4
x_0	2	0	0	0	0	1	-4

← x_5 leaving.

ratio $-\frac{x_0}{x_5}$: $\boxed{-2}$, 0, \, \, \, \

↓ x_1 entering.

(3)	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	0	3	0	1	1	0	2
x_1	1	-2	0	0	-1	0	1
x_3	0	1	1	0	0	-1	4
x_0	0	4	0	0	2	0	-6

Example 2: Sensitivity analysis

Consider the following LPP and its optimal tableau shown below.

$$\text{maximize } x_0 = -2x_1 + x_2 + 2x_3$$

subject to

$$x_1 + x_2 + 4x_3 + x_4 = 9$$
$$5x_1 - x_2 - x_3 + 2x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

The final simplex tableau is given as follows:

Basic	x_1	x_2	x_3	x_4	b
x_4	2	0	2	1	4
x_2	-1	1	3	0	5
x_0	1	0	1	0	15

(a) Suppose $b = (9, 3)^T$ is changed to $\hat{b} = (2, 7)^T$. Find the optimal solution. (You must use dual simplex method.)

Then it becomes:

(1)	x_1	x_2	x_3	x_4	b
x_4	2	0	2	1	3
x_2	-1^*	1	3	0	-1
x_0	1	0	1	0	-1

(2)	x_1	x_2	x_3	x_4	b
x_4	0	2	8	1	1
x_1	1	-1	-3	0	1
x_0	0	1	4	0	-2

(d) Suppose that a new constraint $3x_1 - x_2 - 4x_3 + 3x_4 \leq 5$ is added. Solve the new problem by dual simplex method.

Then it becomes:

(1)	x_1	x_2	x_3	x_4	x_5	b
x_4	2	0	2	1	0	4
x_2	-1	1	3	0	0	5
x_5	3	-1	-4	3	1	5
x_0	1	0	1	0	0	5

(2)	x_1	x_2	x_3	x_4	x_5	b
x_4	2	0	2	1	0	4
x_2	-1	1	3	0	0	5
x_5	-4	0	-7^*	0	1	-2
x_0	1	0	1	0	0	5

$\rightarrow R_2 + (-3) R_1 + R_3$

(3)	x_1	x_2	x_3	x_4	x_5	b
x_4	6/7	0	0	1	2/7	24/7
x_2	-19/7	1	0	0	3/7	29/7
x_3	4/7	0	1	0	-1/7	2/7
x_0	3/7	0	0	0	1/7	33/7

Thus optimal solution is (0,29/7,2/7,24/7).

(b) Suppose $\mathbf{c} = (-2, 1, 2, 0)^T$ is changed to $\hat{\mathbf{c}} = (1, -2, -1, 3)^T$. Use the last simplex table to find the optimal solution.

The new x_0 row is given by

$$\hat{\mathbf{z}}^T - \hat{\mathbf{c}}^T = \hat{\mathbf{c}}_B^T B^{-1} A - \hat{\mathbf{c}}^T$$

(1)	x_1	x_2	x_3	x_4	b
x_4	2	0	2	1	4
x_2	-1	1	3	0	5
x_0	3	0	1	0	12

Solving transportation problem in Excel:

	A	B	C	D	E	F	G	H	I	J
1	Transportation Problem									
2										
3		Unit Cost	Customer 1	Customer 2	Customer 3					
4		Factory 1	40	47	80					
5		Factory 2	72	36	58					
6		Factory 3	24	61	71					
7										
8										
9		Shipments	Customer 1	Customer 2	Customer 3		Total Out		Supply	
10		Factory 1	0	0	0		0	=	100	
11		Factory 2	0	0	0		0	=	200	
12		Factory 3	0	0	0		0	=	300	
13										
14		Total In	0	0	0					
15			=	=	=				Total Cost	
16		Demand	200	200	200				0	
17										
18										

C	D	E	F	G	H	I
Customer 1	Customer 2	Customer 3				
40	47	80				
72	36	58				
24	61	71				
Customer 1	Customer 2	Customer 3		Total Out		Supply
0	0	0		=SUM(C10:E10)	=	100
0	0	0		=SUM(C11:E11)	=	200
0	0	0		=SUM(C12:E12)	=	300
=SUM(C10:C12)	=SUM(D10:D12)	=SUM(E10:E12)				
=	=	=				Total Cost
200	200	200				=SUMPRODUCT(UnitCost,Shipments)